DESIGN AND ANALYSIS OF MARKET PRICES INDICES FOR THE U.S. NATURAL CATASTROPHE EXCESS REINSURANCE TREATIES

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Abstract

This paper proposes several methods for the designing of market price indices for reinsurance excess treaties. The empirical part of the study has been carried out with particular reference to the U.S. market during the period 1975 to 1993, the data coming from a representative sample of nationwide insurance companies. The theoretical part proposes five price indices and one price indicator all being established from the same data. The application of this theory to the U.S. market clearly demonstrates the existence of pricing cycles for the above treaties, linked to important catastrophe claims occurring. Moreover, the econometric analysis allows us to conclude that the frequency of claims is not an intervention variable, but only affects their intensity. Furthermore, this paper supplies a tool for the comparison of a reinsurance company's nationwide portofolio efficiency with that of the market by providing two types of instruments one qualitative and the other structural.

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The paper is divided into five sections

1 Introduction
2 Presentation of the theoretical framework
3 The indices and the indicator : theoretical design
4 Empirical results
5 Modelisation and Econometric Analysis
6 Conclusion

1 Introduction

At the moment, we could consider that it is possible to provide reliable pricing for reinsurance treaties risk and pure premium. With the objective of offering an analysis of the time evolution of the indices, we have chosen in this paper to make an empirical design, from D.G.Friedman's data, and C.Partrat's statistical results (Partrat 1993, Huygues Beaufond and Partrat 1992). But it is important to note that the methodology exposed in this paper also applies when adopting any other premium pricing method, and that the paper provides a general methodology for market price index fixing from any reliable estimation of pure or risk premium of a treaty.

In classical econometric studies of the theory of cycles, prices cannot be observed, thus, it is difficult, when carrying out econometric analysis of observed market result cycles, to distinguish price from insurance supply effect. Through the indices proposed, we can observe the reinsurance market price evolution, and what is more, we can also obtain indications of a part of the market reinsurance behavior evolution with regard to its pricing structure.

We do not attempt to provide here an economic market analysis, but rather to offer an actuarial tool. Because, even if great steps have been made towards solving the premium pricing problem since 1993, reinsurers have not had at their disposal any tool which can both integrate time dynamic into underwriting strategy and proffer a more comprehensive vision of their portofolio. The concept of a price index answers these needs, it is a descriptive measuring instrument for this reinsurance market, and a gauge of company behavior in this regard in terms of efficiency and structure. And, moreover, from a vision of market time evolution, we are able to deduce the opportune short term reinsurance company strategy.

This paper brings to light treaty pricing cycles ; this result was indeed intuitively perceived by market actors, but had not actually been demonstrated until now. The time series econometric analysis of the different indices permits us to test which intervention variables have an impact on actors' behavior. The design of six different indices uncovers the existence of two aspects of actors' behavior implicitly described, that are inherent in reinsurance prices : there is a reaction to catastrophe occurrence operating on the price level, and another which operates on the transformation in the structure of the reinsurance program. The index clearly indicates different underwriting and possible pricing policies according to global market evolution. Finally, it permits the elaboration of a decision strategy concerning longer term commitments.

The most important difficulties which we have encountered fall into four types, and the proposed solutions to these difficulties form the core of the development of this paper. Firstly, from a theoretical viewpoint comes the problem of the choice of the index. From a statistical standpoint, the main difficulty lies in which claims amount distribution to apply. From the actuarial viewpoint, the two principal difficulties concern pricing : firstly, how to take into account the notion of the reinstatement of basic guarantee premium, the conditions of which are concluded at the date of underwriting, and secondly which risk exposure measurement for the insurance companies to choose. Finally, the two principal econometric difficulties are encountered with aggregation and homogenization data problems.

2. Theoretical Framework

The selected method consists in gauging the difference between theoretical and observed prices for each of the years studied. It is based on the postulate that the estimations of the frequency and claim amount distributions are reliable. Consequently, the theoretical prices are taken as a reference.

This choice has been largely determined by constraints imposed and particularly by the available data. We had to imagine a feasible elaboration from the given data, and from the present extent of research in this field. Moreover, the index must obviously be independent from the underwriting policy of the portfolio on which the index is established. This is one of the reasons why it does not seem to be relevant to establish the index solely on the results of Abeille Reassurances, or even on its representative portfolio choice. In another way, if for example, one considers an agregeated index such as (Premiums-Claims)/Premiums, the problem is twofold. First, we do not have available, at a global level, the market results broken down into proportional and non-proportional reinsurance ; and finally, this type of index, usual in reinsurance theory, is not exclusively a price index in as far as it does not allow the differentiation between reinsurance supply and reinsurance price effects.

2.1 Pricing

In practice, reinsurance natural catastrophe treaties bear on all types of undifferentiated natural events. From the statistical viewpoint it is difficult to estimate claim distributions without distinguishing between event types, because in this case, the process is not stationary, and the classical framework of the sampling cannot be applied. The only viable solution then, is estimating claim amount distributions by type of event, and then deducing the compound distribution, or an approximation of the risk premium.
Regarding the results of recent studies (Partrat 1993, Huygues-Beaufond and Partrat 1992) we have chosen the following classification :

(1) Hurricanes
(2) Wind caused events : tornados, storms, floods
(3) Events caused by cold
(4) Earthquakes

The earthquake distribution estimation being not completely satisfactory (because we only have three events in thirty years), we have chosen not to take these claims into account in the pricing ; in fact, this approximation has little bearing on the accuracy of the premiums. It seems, in any case, that the simulation pricing method should give a more reliable theoretical risk premium in this case.

2.2 Distributions

For a given year, and considering the claim type j , $j \in \{1;2;3\}$ we denote
N_j the "annual claim type j frequency" random variable (r.v.),
X_{ij} the "i^{th} claim type j amount of the year" r.v. , $X_j = 0$ and,
S_j the "annual claim type j amount " random variable, thus :

$$S_j = \sum_{i=0}^{N_j} X_{ij}$$

We firstly state the classical pricing assumptions.

A_1 : Claim amounts sampled

For any j , $(X_{ij})_{i \geq 1}$ are independent, identically distributed,

and we call F_j the distribution function, and Xj a r.v. with distribution F_j ("parent variable").

A_2 : The random variables N_j and $(X_{ij})_{i \geq 1}$ are independent, for any j.

The technical assumption being (Huygues-Beaufond and Partrat 1992, Partrat 1993)

A_3 : For any j

N_j is Poisson distributed with parameter λ_j

$$N_j \sim P(\lambda_j)$$

X_j is lognormal distributed with parameters $(\mu_j ; \sigma_j)$

$$X_j \sim LogN(\mu_j ; \sigma_j)$$

with the following numerical values :

Type of claim	**μj** millions US dollars	**σj**	**λj**
1	3.97982	2.48031	2.03
2	3.45598	1.02766	26.44
3	3.39048	1.48864	4.63

A4 : Independence between type of risks

We must notice here that it is possible, in the same way, to study the case in which the random variable X_j is Pareto distributed with parameters depending on the priority.

2.3 Global Pricing for the Market

2.3.1 Pure Premium

Let us consider the Xs reinsurance treaty with priority M_1 and limit M_2 , and admit an unlimited number of free reinstatements of the basic guarantee. And firstly take an insurance company representating all the market which should bear the total amount of each claim occurrring.

We will denote :

$C_{ij}(M_1;M_2)$, the claim amount borne by the reinsurer for type j coverage for the claim amount X_{ij} .

By definition, $C_{ij}(M_1;M_2) = 0$, if $X_{ij} \leq M_1$

$= X_{ij} - M_1$, if $M_1 \leq X_{ij} \leq M_2$

$= M_2 - M_1$, if $X_{ij} \geq M_2$

$C_j(M_1;M_2)$, the parent variable

$S_j(M_1;M_2)$, the aggregate amount of type j claims borne by the reinsurer

$$S_j(M_1;M_2) = \sum_{i=0}^{N_j} C_{ij}(M_1;M_2)$$

$S(M_1;M_2) = \sum_{j=1}^{3} S_j(M_1;M_2)$ the aggregate amount of all type claims

$P_j(M_1;M_2)$, the corresponding pure premium,

$P_j(M_1;M_2) = E[S_j(M_1;M_2)]$

$= \lambda_j \; E[C_j(M_1;M_2)]$, where the expectation is calculated under the theoretical probability claim type j occurrring

$$= \lambda_j \left[\int_{M_1}^{M_2} (x - M_1) dF_j(x) + (M_2 - M_1)(1 - F_j(M_2)) \right]$$

The total pure premium being :

$$P(M_1;M_2) = E[S(M_1;M_2)] = \sum_{j=1}^{3} P_j(M_1;M_2)$$

We call Φ the standard Normal distribution function, thence :

$$F_j(x) = \Phi\left(\frac{\ln x - \mu_j}{\sigma_j}\right)$$

Proposition 1 :

The pure premium can then be written :

$$P_j(M_1;M_2) = \lambda_j\, E[C_j(M_1;M_2)] \quad \text{with (classical results),}$$

$$E[C_j(M_1;M_2)]$$
$$= e^{\mu_j + \sigma_j^2/2}\left[\Phi\left(\frac{\ln M_2 - \mu_j}{\sigma_j} - \sigma_j\right) - \Phi\left(\frac{\ln M_1 - \mu_j}{\sigma_j} - \sigma_j\right)\right] - M_1\left[1 - \Phi\left(\frac{\ln M_1 - \mu_j}{\sigma_j}\right)\right] + M_2\left[1 - \Phi\left(\frac{\ln M_2 - \mu_j}{\sigma_j}\right)\right]$$

2.3.2 Risk Premium

The considered loading will be proportional to the pure premium (expected value principle) or to the standard deviation (standard deviation principle) of the amount of claims borne by the reinsurer variable. This kind of loading allows us to integrate the marked variance of the process in the compensation risk which is supported by the reinsurer.

But the theoretical evaluation does not take into account broker costs, these rates being different whether they concern either the basic guarantee or the reinstatement premiums. We notice that these risk premium calculation principles possess, moreover, properties useful for monetary exchanges (Goovaerts et al 1990, Gerber 1979).

For the expected value principle, the risk premium will be :

$$\Pi(M_1;M_2) = \sum_{j=1}^{3} \Pi_j(M_1;M_2)$$

with $\Pi_j(M_1;M_2) = (1+\eta)P_j(M_1;M_2)$

so that : $\Pi(M_1;M_2) = (1+\eta)P(M_1;M_2)$

For the standard deviation principle, the risk premium for type j claim can be written :

$$\Pi_j(M_1;M_2) = E[S_j(M_1;M_2)] + \eta\sigma[S_j(M_1;M_2)]$$

$$= \lambda_j E(C_j(M_1;M_2)) + \eta\left(\lambda_j E\left[C_j(M_1;M_2)^2\right]\right)^{1/2}$$

Denote, $w_j(M_1 : M_2) = E\left[C_j(M_1;M_2)^2\right]$, we have :

$$w_j(M_1;M_2) = \int_{M_1}^{M_2} x^2 dF_j(x) - 2M_1\int_{M_1}^{M_2} x dF_j(x) - M_1^2[F_j(M_2) - F_j(M_1)] - (M_2 - M_1)^2[1 - F_j(M_2)]$$

So, the risk premium will be :

$$\Pi(M_1;M_2) = P(M_1;M_2) + \eta\sigma(S(M_1;M_2))$$
$$= P(M_1;M_2) + \eta\sqrt{\sum_{j=1}^{3} \lambda_j w_j(M_1;M_2)}$$

In the case where F_j is a Lognormal distribution function, the obtained result can be stated:

$$w_j(M_1;M_2) = e^{2(\mu_j+\frac{\sigma_j^2}{2})}\left[\Phi\left(\frac{\ln M_2-\mu_j}{\sigma_j} - 2\sigma_j\right) - \Phi\left(\frac{\ln M_1-\mu_j}{\sigma_j} - 2\sigma_j\right)\right] - 2M_1 e^{(\mu_j+\frac{\sigma_j^2}{2})}\left[\Phi\left(\frac{\ln M_2-\mu_j}{\sigma_j} - \sigma_j\right) - \Phi\left(\frac{\ln M_1-\mu_j}{\sigma_j} - \sigma_j\right)\right]$$
$$+ M_1^2\left[\Phi\left(\frac{\ln M_2-\mu_j}{\sigma_j}\right) - \Phi\left(\frac{\ln M_1-\mu_j}{\sigma_j}\right)\right] + (M_2 - M_1)^2\left[1 - \Phi\left(\frac{\ln M_2-\mu_j}{\sigma_j}\right)\right]$$

A5 : We will take as the theoretical risk premium the sum of all the risk premiums for the different claim types.

Meaning that we will take $\sum_{j=1}^{3} \Pi_j(M_1;M_2)$ instead of $\Pi(M_1;M_2)$

It may be noted that in the expected value principle pricing case, this assumption is fulfilled. Whereas, in the standard deviation principle pricing case, this choice is not completely satisfactory. However, it will permit us, firstly, to consider only one claim type and thus, we will try to highlight methodological problems, and secondly we will construct the aggregated claims index. Moreover, the empirical results are close to this approximation.

Proposition 2 :

Under the assumptions A1 to A5, the theoretical risk premium is overvalued whilst not imperilling the survival of the reinsurance company.

Indeed, for the standard deviation principle, we have :

$$\Pi(M_1;M_2) \leq \sum_{j=1}^{3} \Pi_j(M_1;M_2)$$

2.4 Pricing for one Given Insurance Company

For more details, and specially for the demonstrations concerning this part we refer the reader to previous studies (Cadinot et al 1991, Lion 1993).

2.4.1 Insurance Company Risk Measure

The specific risk associated with natural catastrophes reinsurance lies in the insurance companies' geographic exposure. With statistical exactness, the insurance company Xs premium should be obtained from the company's historical estimated parameters. Two problems then arise : the information provided by the companies varies which could necessitate specific study by company, and furthermore, the company portfolio period evolution and an anticipation of its underwriting policy, should necessarily be known for an accurate updating of claim costs. The proposed solution privileges the global market data claims, and, to an insurance company, a risk exposure measure.

Still considering the market at a fixed date, and in order to propose measures for the insurance companies both homogeneous and consistent with insurance branches affected by the covered risks, we have chosen to define for each claim type, a "natural catastrophe premium" corresponding to each insurance company or to the whole market. The chosen weighting corresponds with one of those used by the market, the claim type geographic zoning coming from the country subdivision into six exposure areas. The principal assumption underlying the method signifies that insurance company portfolios are supposed to be stable during the year for the earned or written annual premium breakdowns to be considered as equal.

Definition : (Best Review)

For the insurance company c, we can define the catastrophe premium incomes (CPI) for the claim type j as :

$$0.03\, Fi_j^{(c)} + 0.03\, IM_j^{(c)} + 0.5\, AL_j^{(c)} + 0.25(H_j^{(c)} + F_j^{(c)}) + 0.12\, CMP_j^{(c)} + 0.02APD_j^{(c)}$$

with the following notations,

3% Fire Premium Incomes	0.03	Fi
3% Inland Marine Premium Incomes	0.03	IM
50% Allied Lines Premium Incomes	0.5	AL
25% Homeowners Premium Incomes	0.25	H
25% Farmowners Premium Incomes	0.25	F
12% Commercial Multiperil Premium Incomes	0.12	CMP
2% Auto Physical Damage Premium Incomes	0.02	APD

The events exposure type j gauge of the insurance company c is measured by $q_j^{(c)}$ defined as :

$$\frac{0.03\, Fi_j^{(c)} + 0.03\, IM_j^{(c)} + 0.5\, AL_j^{(c)} + 0.25(H_j^{(c)} + F_j^{(c)}) + 0.12\, CMP_j^{(c)} + 0.02APD_j^{(c)}}{0.03\, Fi_j + 0.03\, IM_j + 0.5\, AL_j + 0.25(H_j + F_j) + 0.12\, CMP_j + 0.02APD_j}$$

The numerator represents the CPI amount for the type j and the insurance company c. The denominator represents the CPI amount for the type j and the entire market. So, this risk type j measure can be interpreted as a " catastrophic type j company market share".

A6 : For $j \in \{1;2;3\}$ and, for $c \in C, q_j^{(c)} \neq 0$; where C denotes the insurance company set.

2.4.2 Insurance Company Claims Distribution

Definition :

For a given year, the cost of the i^{th} claim charged to the insurance company c, for the claim type j will be defined as :

$$X_{ij}^{(c)} = q_j^{(c)}\, X_{ij}$$

Proposition 3 :

The frequency claim type j for the insurance company c random variable is equal to N_j.

The amount of claims borne by the insurance company c random variable is distributed according to the Lognormal distribution deduced from those of X_{ij} by a mean translation of $\ln q_j^{(c)}$.

That is :

$$X_{ij}^{(c)} \sim LogN(\mu_j + \ln q_j^{(c)}; \sigma_j)$$

According to the assumption A4, it is only necessary to establish the Xs treaty pricing method for one claim type. So, we present the result as a proposition.

Proposition 4 :

The pure premium of the insurance company c and the risk type j is :

$P_j^{(c)}(M_1;M_2)= q_j^{(c)} P_j(M_{1j}^{(c)};M_{2j}^{(c)})$

where, $M_{ij}^{(c)} = M_i/q_j^{(c)}$, for i belonging to {1;2}

The pure premium loading is :

$w_j^{(c)}(M_1;M_2)= q_j^{(c)2} w_j(M_{1j}^{(c)};M_{2j}^{(c)})$

The risk premium is :

$\Pi_j^{(c)}(M_1;M_2)= q_j^{(c)} \Pi_j(M_{1j}^{(c)};M_{2j}^{(c)})$

By this pricing method, the insurance company risk exposure is measured by an "insurance company considered type risk share" factor. The insurance company premium for a treaty b Xs a, is (except on the factor) the corresponding premium of the insurance company taken as representative of the whole market for the treaty in which the priority and the limit have been "dilated" by the inverse of the insurance company "risk share".

3. Indices and Indicator : Theoretical Design

Assuming that we know a reliable U.S. natural catastrophe Xs treaties pure premium evaluation, this part of the paper proposes five price indices that can be grouped in three families. The first family definies a market loading rate, the second one compares the "loaded observed premium on theoretical risk premium" rate to one, and the third compares the observed to the theoretical insurance company threshold means. The theoretical econometric difficulty here lies in the data aggregation method choice. How can a market price index be established from program layer data of each insurance company ? It appears practically impossible to find a transformation permitting the homogenization of insurance companies. This is why the proposed solution lies firstly, in aggregating each program's data and secondly, the insurance companies' data.

3.1 Framework

At this stage, we will assume as known :

the reinsurance programs for n years and for the $\mathcal{C}$ of insurance company set (premium rates and insurance company realised incomes),

the risk measures by claim type and by insurance company for the different years,

the CPI for each year, each insurance company, and each claim type.

Furthermore, we are able to compute the theoretical premiums, the loading factor, and the theoretical risk premiums for each insurance company[1].

This part is elaborated for a fixed date.

Given c, an insurance company belonging to $\mathcal{C}$, its reinsurance program will be represented by $K^{(c)}$ Xs layers index-linked by $k \in \{1;...K^{(c)}\}$; its corresponding priorities and limits will respectively be denoted $\left\{(M_{1k}^{(c)};M_{2k}^{(c)})\right\}_{k\in\{1;..K^{(c)}\}}$; the observed loaded premiums, and theoretical risks premiums, of the respective layers will be denoted as $\left(\Pi_k^{(c)obs}\right)_{k\in\{1,..;K^{(c)}\}}$ and $\left(\Pi_k^{(c)th}\right)_{k\in\{1,..;K^{(c)}\}}$.

In any case, we postulate :

[1] Abeille Réassurances software.

Postulate :

Theoretical risk premiums are accurately evaluated for each treaty.

3.2 First Proposed Index : Loading rate Standard Deviation Principle

We consider here the theoretical risk premium evaluated by standard deviation principle, and we take its risk loading rate, η as parameter. The theoretical risk premium for the insurance company k^{th} layer is written as the η variable function :

$$\Pi_k^{(c)^{th}}(\eta) = P_k^{(c)} + \eta\sigma_k^{(c)}$$

3.2.1 Insurance Company c Loading Rate

Proposition and Definition 5 :

(i) Define the insurance company c loading rate by :

$$\eta^{(c)} = \underset{\eta}{\mathrm{Arg\,Min}} \sum_{k=1}^{K^{(c)}} \frac{\left[\Pi_k^{(c)^{th}}(\eta) - \Pi_k^{(c)^{obs}}\right]^2}{\Pi_k^{(c)^{obs}}}$$

(ii) $\eta^{(c)}$ exists and is unique.

Proof : (ii) comes from classical result of projection theorem in a closed set of $\mathbf{R}^{K^{(c)}}$.

At a given date, and for a given insurance company c, $\eta^{(c)}$can be interpreted as the premium loading rate which permits the best adjustment, in the distance sense above, of the $K^{(c)}$ theoretical risk premiums, to the observed loaded premiums (assimilated to market price) of the program for c.

For a given insurance company of the reinsurer portfolio, it is then possible to estimate its "theoretical administrative costs" (including risk compensation) and thus, to compare it with the real costs. But this must be interpreted carefully.

Indeed, suppose that we know the real reinsurer loading rate for c, $\eta^{(cR)}$ and, suppose, for instance that $\eta^{(cR)} < \eta^{(c)}$.

It could do because the reinsurer only keeps in its portfolio the less exposed part of the insurance company program, and thus the mean standard deviation is higher than that of the market or, that the reinsurer has fewer costs than the market. It could be worthwhile elaborating reinsurer efficiency analysis. For this, if we consider that the entire program is selected by the reinsurer, the two rates are comparable, if not, then we can compute the market equivalent rate with the corresponding layers of the program, thence analysing reinsurer strategy in regard to the market.

Proposition 6 : $\eta^{(c)}$ Explicit Formula

$$\eta^{(c)} = \frac{\sum_{k=1}^{K^{(c)}} \sigma_k^{(c)^{th}} \left(\Pi_k^{(c)^{obs}} - P_k^{(c)^{th}}\right) \Big/ \Pi_k^{(c)^{obs}}}{\sum_{k=1}^{K^{(c)}} \left(\sigma_k^{(c)^{th}}\right)^2 \Big/ \Pi_k^{(c)^{obs}}}$$

This result is an immediate consequence of the previous proposition.

3.2.2 Market Loading rate

In order to aggregate insurance companies and to establish the market index price, we must at this stage provide an equivalent measure for each insurance company.
Note that heterogeneities of insurance company premium assessment are actually rectified by program premium rates. Thence, we cannot take premium assessments as comparable mesures among insurance companies. The paper proposes a homogeneous measure for each insurance company c, $P^{(c)}$, being the result of the computation of the CPI for the company c all over the U.S..

Definition :

The market price index will be defined as :

$$I_1 = \frac{\sum_{c \in C} \eta^{(c)} P^{(c)}}{\sum_{c \in C} P^{(c)}}$$

Notice that I_1 realizes the indices elementary properties : it is a homogeneous and growing function of $(\eta^{(c)})_{c \in C}$ vector.
At a fixed date, I_1 can be considered as the market "mean" loading rate for the standard deviation principle evaluation. We are now able to measure the reinsurer efficiency within the market, on the part of its portfolio whose exposure type is comparable with that of the portfolio used for market index contruction. The reinsurer can have a lesser rate than that of the market for several reasons as seen above in the insurance company index case.

3.3 Second Price Index : Pure Premium Proportional Loading rate

The previous elaboration is repeated, but the theoretical risk premium pricing is calculated using the expected value principle and shall be written as the variable α function :

$$\Pi_k^{(c)^{th}}(\alpha) = (1+\alpha) P_k^{(c)}$$

Keeping the same notations, the market rate for the insurance company c is now written :

$$\alpha^{(c)} = \frac{\sum_{k=1}^{K^{(c)}} P_k^{(c)^{th}}}{\sum_{k=1}^{K^{(c)}} \left(P_k^{(c)^{th}}\right)^2 \Big/ \Pi_k^{(c)^{obs}}} - 1$$

And, as seen above, the market price index I_2, is defined from insurance companies aggregation by the weighting of their respectives loading rates with their catastrophe premiums.

Definition :

Price index market will be defined as :

$$I_2 = \frac{\sum_{c \in C} \alpha^{(c)} P^{(c)}}{\sum_{c \in C} P^{(c)}}$$

This index can be seen as a loading rate defined by the market, but within the framework of the expected value principle. In principle, the index values should be higher in this case. This is because, for natural catastrophe events, the occurrrence of claims being rare, the standard deviation is much greater than the mean value. This index is by definition less sensitive to claim amount variance and thus it hardly takes into account the pricing spread.

3.4 Third Index : Quotient of Two Premiums

From the first index time evolution observation, set out in the next section, and from the confrontation of the results with market reinsurers practice, it seems reasonable to fix a loading rate of 20% within standard deviation principle framework. Notice that this result is only useful for the nationwide U.S. market.
We can then define the theoretical (resp. observed) insurance company c aggregated premium as:

$$\Pi a^{(c)^{th}} = \sum_{k=1}^{K^{(c)}} \Pi_k^{(c)^{th}} \quad \left(\text{resp. } \Pi a^{(c)^{obs}} = \sum_{k=1}^{K^{(c)}} \Pi_k^{(c)^{obs}} \right)$$

It is to be remembered that each theoretical risk premium depends on the insurance company by its measure of risk. Moreover, notice that, in the present case, the premium is no longer a function, but a numerical value, thus, the aggregated theoretical premium is the insurance company theoretical program price.
In order to aggregate the insurance companies, we need only to add their respective theoretical and observed associated premiums which will be supposed representative of the market.

Denote : $\Pi^{obs} = \sum_{c \in C} \Pi a^{(c)^{obs}}$ and $\Pi^{th} = \sum_{c \in C} \Pi a^{(c)^{th}}$

Definition :

The market index price will be defined as : $I_3 = \dfrac{\Pi^{obs}}{\Pi^{th}}$

The interpretation has to take into account the fact that the right price is given by the index value 1. A reinsurance company having the corresponding index lesser than the observed market level, and greater than one, is more efficient than the market (and vice versa).

3.5 Forth Indicator : Evolution of "Centroids"

This idea is more abstract, however the principle used remains essentially the same. Nevertheless, we must notice that the matter here concerns a price indicator of the U.S. Xs treaties time evolution pricing, and not a price index. The underlying idea consists in studying the structural evolution of the programs ; i.e. the possible balance or imbalance between high and low program layers which operate in time. To this end, we compute theoretical and observed centroids, both being defined by the market portfolio as follows.

Proposition 7:

Given c, an insurance company, for any value of k element of $\{1;K^{(c)}\}$, a treaty of the program , there exists one and one only point $G_k^{(c)}$, element of $\left[M_{1k}^{(c)} \; ; \; M_{2k}^{(c)} \right]$, centroid of the treaty defined as:

$$\int_{M_{1k}}^{G_k} dF(x) = \int_{G_k}^{M_{2k}} dF(x)$$

where F is the claims amount distribution function.
Another way of stating this, is that for a given insurance company, and a given program layer, there exists one unique centroid of the class for the claims probability measure.

Consider now, always for a given insurance company, the following applications :

$$\{1;..;K^{(c)}\}\xrightarrow{\tau^{(c)}}\mathbf{R}^2 \qquad\text{and}\qquad \{1;..;K^{(c)}\}\xrightarrow{\zeta^{(c)}}\mathbf{R}^2$$

$$k \longrightarrow (G_k^{(c)};\Pi_k^{(c)^{th}}) \qquad\qquad k \longrightarrow (G_k^{(c)};\Pi_k^{(c)^{obs}})$$

Thus, to each program layer k, can be associated two couples : one theoretical, and one observed. The theoretical couple is the treaty's centroid weighted by the theoretical premium defined above ; the observed couple is the same theoretical centroid, but weighted by the real observed premium. Proceeding with this method, it is possible to associate with each insurance company, two unique couples (theoretical and observed) of centroids weighted by their respective coefficients. Indeed, for any portfolio insurance company c, there exists $K^{(c)}$ respectively theoretical and observed couples as defined above.

Centroid $G^{(c)^{th}}$ (resp. $G^{(c)^{obs}}$) corresponding to the insurance company c, is defined by the barycenter of $\left\{\left(G_k^{(c)};\Pi_k^{(c)^{th}}\right)_{k\in\{1;..;K^{(c)}\}}\right\}$, resp. $\left\{\left(G_k^{(c)};\Pi_k^{(c)^{obs}}\right)_{k\in\{1;..;K^{(c)}\}}\right\}$.

The weightings corresponding to the insurance company c are respectively the theoretical or the observed premium sums. Finally, the market representation points, denoted by G^{obs} and G^{th}, are defined in the same way, as barycenters respectively theoretical and observed of the set of couples of centroids corresponding to insurance companies.

Definition :

The treaties pricing structure indicator is defined by the algebraic measure, updated to mid-1992 :

$$I_4 = r_n . \overline{G^{th}G^{obs}}$$

where, r_n is the updating factor of the claim amounts for year n to the mid-1992 value.

Here, the reinsurer corresponding indicator level should be positive and lower than that of the market to describe prices that are more homogeneous with theoretical prices and, thus, economically more efficient.

3.6 Fifth Index : Centroids Quotient

Working from the previous elaboration, we propose here an index more convenient for business underwriters, comparable to one and which has the advantage of homogeneity.

Definition :

$$I_5 = \frac{\overline{OG^{obs}}}{\overline{OG^{th}}}$$

When I_5 differs greatly from one, it induces a pricing structure lacking homogeneity between observed and theoretical pricing. Nevertheless, this homogeneity can result either from practiced prices or from the program structure.

These indices provide economic tools for the shadowing of the U.S. natural catastrophe reinsurance Xs treaty market prices. Apart from the analysis and the statement of eventual market cycles observed by the actors, two functional applications can quickly be considered for a reinsurance company. The first is an indication and aid to underwriting decision, which can be provided thanks to the econometric study of one or more indices. The second consists in giving indications on reinsurance company efficiency, and it requires not only supplementary work on the company's portfolio, but also a mastery of the market for interpretation and portfolio choice.

4. Empirical Study

4.1 Time Dynamic Introduction

The design of a time evolution price index needs the adoption of an update method. We have considered the same pricing method whatever the year, in order to avoid the psychological bias due to the impact of the claims known prior to the index evaluation date.
Of the two methods which can be envisaged (claim costs update, or reinsurance premiums update), we have selected the former. Factors, obtained from D.G. Friedman's data base, give a mean annual "inflation" rate of 4 % which seems a low value for it should take into account all economic factors considered in the process. This is why, the paper proposes to increase the factors by 3 % each year. The respective years price indices are established with rates indicated in the table below.

Claim Type Year	**1 3%**	**2**	**3 of supplementary annual inflation**	**4**	**Mean rate**
1975	0.240	0.270	0.273	0.270	0.264
1976	0.307	0.300	0.327	0.300	0.308
1977	0.327	0.339	0.352	0.339	0.339
1978	0.340	0.373	0.407	0.373	0.373
1979	0.353	0.402	0.422	0.387	0.391
1980	0.367	0.446	0.459	0.440	0.428
1981	0.428	0.480	0.512	0.474	0.474
1982	0.489	0.516	0.530	0.510	0.511
1983	0.514	0.562	0.585	0.547	0.552
1984	0.625	0.613	0.610	0.595	0.611
1985	0.665	0.662	0.672	0.635	0.659
1986	0.676	0.695	0.688	0.678	0.684
1987	0.719	0.734	0.739	0.723	0.729
1988	0.765	0.774	0.782	0.775	0.774
1989	0.828	0.834	0.834	0.832	0.832
1990	0.896	0.896	0.896	0.892	0.895
1991	0.957	0.957	0.957	0.957	0.957
1992	1	1	1	1	1
1993	1.045	1.045	1.045	1.045	1.045

The time dynamic introduction, imposes only to be interested in "claim amount" random variable transformation for each considered event type, because the catastrophe events occurring frequency is independent of transformations on the economy. In order to be able to use the most reliable statistics, it is preferable to take into account the parameters estimated on claims until 1992. To permit this, we need to deduce the year n claim cost distribution, from the starting assumption bearing on year 1992 costs.
Denote : X_n the year n claim costs random parent variable and, r_n, the year n claim costs update factor assigned to 1992 costs.

Proposition 8:

Lognormal distribution case :

$$\text{if } X_{1992} \sim \text{LogN}(\mu ; \sigma) \text{ then, } X_n \sim \text{LogN}(\mu + \text{Log}r_n ; \sigma)$$

Pareto distribution case :

$$\text{if } X_{1992} \sim P(b;\theta) \text{ then, } X_n \sim P(br_n ;\theta)$$

where, P(b;t) indicates Pareto distribution whose distribution function is given by :

$$F(x) = [1-(b/x)^{\theta}]\, 1\!\!1_{]b;+\infty[}(x)$$

4.2 Data Homogenization

4.2.1 Coherence Between Observed and Theoretical Data

Assumption :

In the elaboration of the proposed indices, we assume that cost of claim and frequency distributions are known, hence, the theoretical pure premium is assumed to be properly adjusted.

In practice, observed premium can be decomposed into the sum of the basic guarantee hedging premium and the basic guarantee reinstatement premium when an event occurrs; and this leaving aside broker costs.

The theoretical risk premium computed takes into account unlimited losses occurrring for the insurance company, and whose amount should be included between the priority and the limit of the treaty. On the other hand we have at our disposal the loaded premium observations which are paid by insurance companies at the end of each exercise, for each treaty of each program of the basic guarantee, out of reinstatement guarantee. Now, reinsurance Xs treaty practiced prices involve both the premium rate applied to a given assessment (generally the direct insurance company commercial premium) and the particular conditions for the guarantee reinstatement, which depend on the losses borne by insurance company during the given exercise. These two points are an integral part of the treaty price at the signature of the aggreement. To be consistent, we should make a rectification for each basic observed guarantee premium by weighting it with the probability of successive occurrences of one, two, etc.. until an infinite number of claims included in the Xs layer, and this in order to elaborate the most possible accurate index, and to make both premiums (theoretical and observed) comparative.

Nevertheless, we do not agree with this solution because it does not take into account the implicit program pricing structure of reinstatement conditions. We have then chosen to make here a reinstatement extra premium approximation, founded on the basic guarantee fixed price and the contract guarantee reinstatement conditions.

Take an insurance company c and a reinsurance program treaty k. And denote P_{obs} , the premium paid up for the treaty basic guarantee.

Assumption :

For any year, and any treaty, we subsequently assume that each treaty has a probability of one to a total loss to the layer, and to include one basic guarantee reinstatement.

This imposes a rectification on each treaty price, in order to refer to an equivalent treaty but which should have one, and only one, free principal guarantee reinstatement. This correction must moreover, take into account practiced market reinstatement conditions, that is : generally at 100 % prorata capita, and or proprata temporis, or without prorata temporis (for the recent years), or finally with other conditions over the period. This assumption, credible today, is less conceivable for previous years, nevertheless, it is pertinent to adopt it, regarding time homogeneity of the analysis and in order to integrate low and high program layers differenciation. The question now is how from this information to estimate a mean reinstatement premium fitting observed prices from basic pricing. The solution proposed here consists in considering a global uniform methodology for treaties and insurance companies.

Take, for each layer, the threshold G, centroid of the layer.

Postulate :

We assume subsequently that the reinstatement premium amount of the treaty whose basic premium is denoted by P_{obs} , and whose centroid is denoted by G, is computed as :

$$P_{obs}\, P(N(G) \geq 1)$$

where N(G) is the parent r. v. assigning the number of claims whose amount is higher than G, This extra premium estimation fits a mean premium basic guarantee reinstatement hedging price.

Consequently, we will define the treaty observed loaded premium, as :

$$\Pi_{obs} = P_{obs}(1+ a\, P(N(G) \geq 1))$$

where a is the mean factor fitting renewal conditions, that is :

a =1 in the case of without prorata temporis reinstatement
a=0.5 in the case of prorata temporis reinstatement
a=0.25 in the case of 25 % reinstatement

Hence : $\Pi_{obs} = P_{obs}\,[1+ a - a.\exp(-\lambda(1-F(G))]$

This result comes from :

$$N(G) \sim P(\lambda(G)) \text{ , où } \lambda(G) = \lambda\,(1-F(G)),$$

with, λ the frequency for the given claim type parameter.

Notice that for the practical index elaboration, the value of the considered extra premium is obtained as the mean value of the three reinstatement extra premiums associated to the three claim types.

4.2.2 Empirical Measure Computations

We want to underline here only one point concerning the empirical part of the paper concerning the centroid computations at a given date. Theoretically for a given claim type the treaty's centroid computation does not pose any problem, practically the difficulty lies in the hedging of all claim types by the agreement. The adopted method here consists for each treaty in determining the centroid for each claim type first by obtaining the three mean thresholds associated with the insurance company treaty, and afterwards the treaty centroid is defined as the mean value weighted by the insurance company risk measures for the different claim types.

4.3 Two Types of Index Elaboration

Two types of indicator have been elaborated one without rectifying the real reinsurance observed premiums and the other with the above indicated rectification. The second type maybe permits a more flattering interpretation for a presentation concerning for example market rentability. The first possesses fewer approximations, and shall be chosen for econometric analysis.

4.4 Data Presentation

The data base study support comes from the joint portfolio of l'Abeille Réassurances and l'Union Française de Réassurances. The work is thus based on a sample of twelve insurance companies nationwide, for which it could have been possible to reconstruct complete files from 1975 to 1993. Insurance company sizes vary and in relation to a theoretical nationwide sample. Ours possesses a slight North-East U.S. over-exposure. We can notice that their risk exposure measure[2] time evolution is not stable. The risk exposure measures have been computed for each insurance company and each year from the states and insurance branch break down of the company and of the U.S.. This work has been carried out over 18 years. As the 1993 breakdown is presently unknown we have taken the 1992 measurements for the elaboration of the index. For each year it has been possible to list the whole programs. The treaties take effect either in January and then the treaty exercise coincides with the index computation exercise year, or in the first part of the year and then the index computation is made for the effective date year, or finally, it takes effect in the second part of the year and

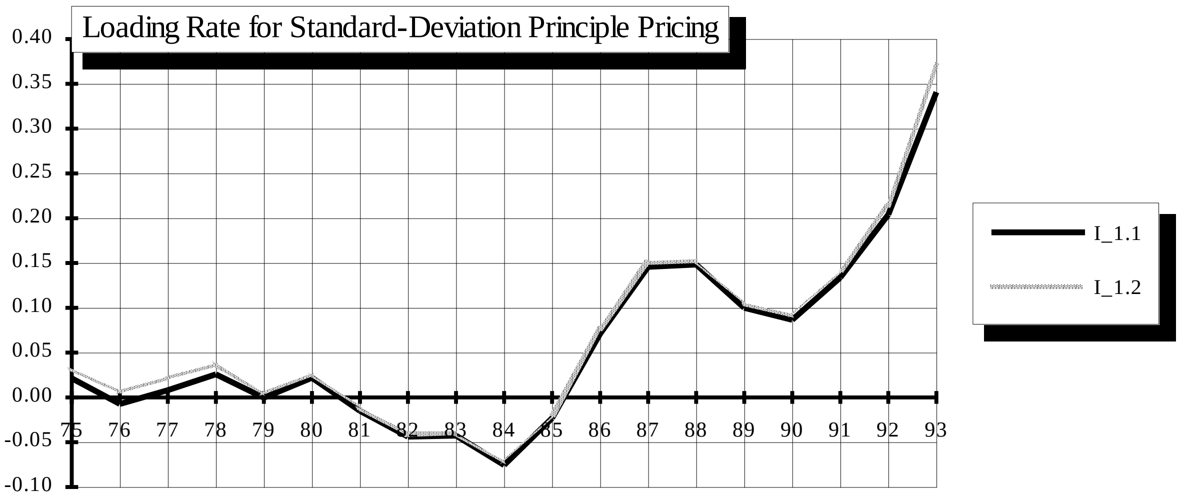
[2]This data is not published here for confidential reasons.

then the index is computed for the following effective date year. The sample includes three insurance companies of late effective date, and three of early ones. Note at this stage that the approximation made consists in assuming a short term stability of the insurance company portfolio in order to keep the computation of its risk exposure measure estimations coherent. With the programs have been listed the guarantee reinstatement conditions variable according to insurance companies years and treaties. The basic file is compound of 25 columns and 1388 lines giving for each treaty the market data (claim type updating factors) and the insurance company data (treaty, reinstatement conditions, premium rate, premium assessment realised at maturity, effective date, claim type risk exposure measure, CPI). We have then, from this basic data prepared another intermediate file for the computation of the annual indices. The theoretical prices have been computed under the assumption A3 and the study can be extended to Poisson compound Pareto distributions. The theoretical pure premiums, loadings, centroids, and rectified observed premiums have been computed for each treaty. The last step is to compute each desired index or indicator.

4.5 Evolution Curves

We present two curves for each index, one with the observed premiums without rectification (indexed by .1) ,the other with rectified observed premiums (indexed by .2)

4.5.1 First Index Family



First of all, notice that the comparison of the two curves indicates that the observed premiums adjustment hardly affects the shape in its convexity, slightly increasing the index obtained over initial and recent years. For the latest years this result proceeds from the guarantee reinstatement extra-premium market pricing upswings from 1991. Three pricing periods are clearly shown by these curves :

1975-1984 with a clear risk undervaluation
1984-1990 with a mean loading rate of 5 % of the standard deviation
1990-1993 with a risk compensation market requirement.

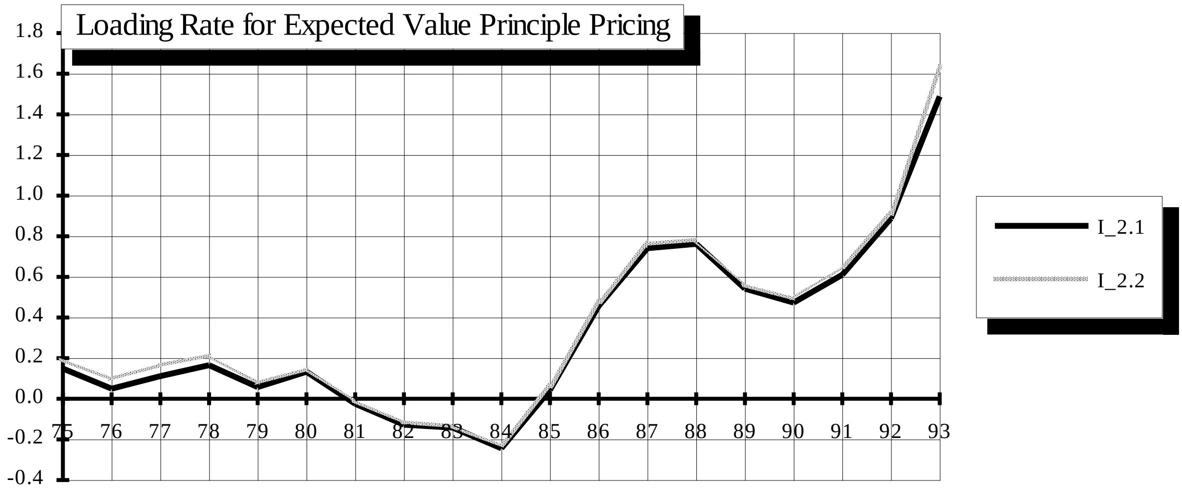
Moreover four periods showing actors' behavior cycles can be distinguished from the shape of the curves.

1975-1983 with a market loading rate close to zero, or even negative, tending to prove that the reinsurers' covered risk was undervalued, and did not take into account the claim amounts variance.

1984-1986 during this period treaty prices rose to a threshold of 15% of the standard deviation, at the end of the period. This change in level can be explained by the occurrence of hurricane Alicia in 1983. We notice that the market reacted two years after the claim date.

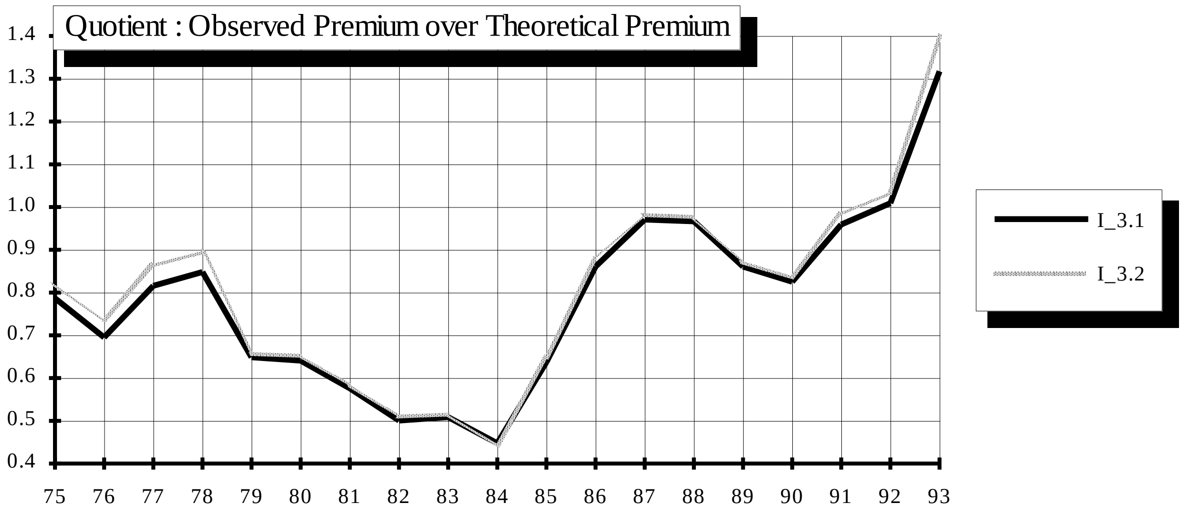
1987-1989 prices fell due to the absence of claims during the period

From 1990 we can observe a new rise in prices seeming to bring about a new mean rate of risk compensation between 20 % and 35 % of the standard deviation for I_1.1 and 25% and 40% for I_1.2. This latest and significant reaction which permits reinsurance companies to reach a real risk compensation level, is probably due to the simultaneous effects of the important and frequent claims occurring and the market financial capacity contraction. Finally, we can conclude as assumed, that a rate of 20 % is relevant for risk premium loading in standard deviation principle pricing.



Obviously, the shape of these two curves is likely to be comparable to previous ones. This tends to prove that market reactions in this kind of reinsurance depends on the mean level of insured risk and not on its variability. Evidently, the degree of size of the rates is higher here because in the case of rare events such as natural catastrophes the standard deviation is much greater than the random variable expectation.

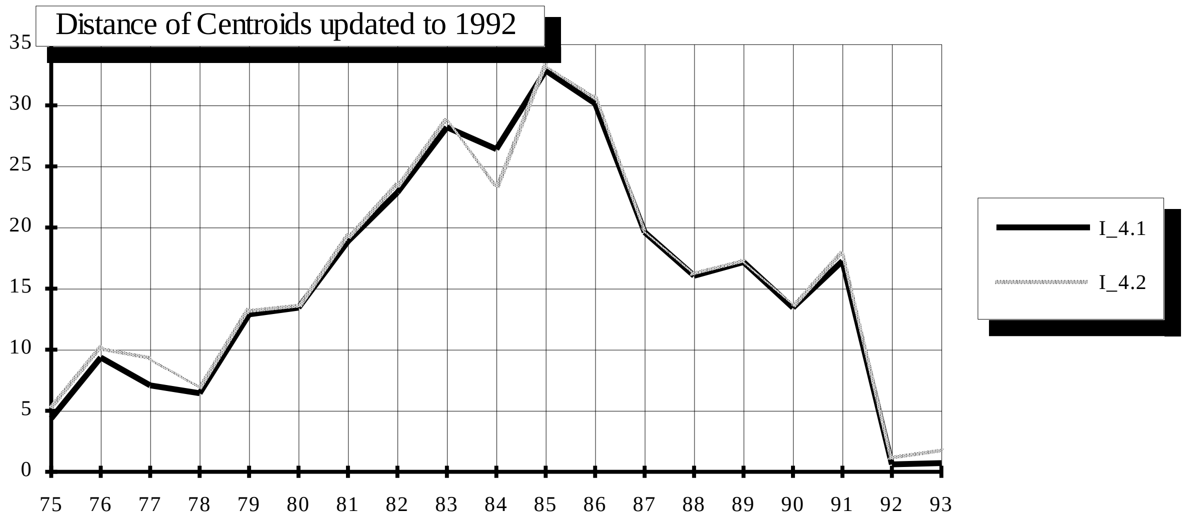
4.5.2 Second Family Indices



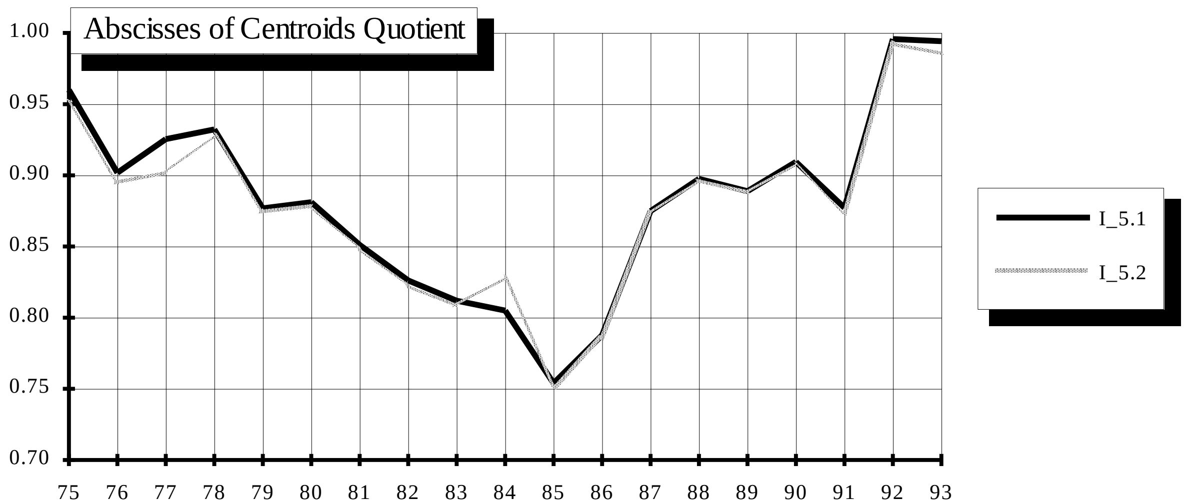
These two curves indicate the same pricing cycles as those of the previous family, but with more accentuated evolutions.
From 1990 the occurrence of hurricane Hugo implies a new rise thus permitting the attainment of a sufficient adjustment threshold. But we must modulate this appreciation by noting the fact that it only takes into account the occurrence of the sole major claim Andrew which reactivated with only one year's delay reinsurance premiums occurring in 1993, and this for the first time. Notice at this stage that it is difficult to differentiate the reasons for this

great increase. Indeed, it is impossible to know whether it results from the two year interval succession of two major claims, or solely from the extent of claim Andrew, or finally from the simultaneity of these events with contraction of the market financial capacity.

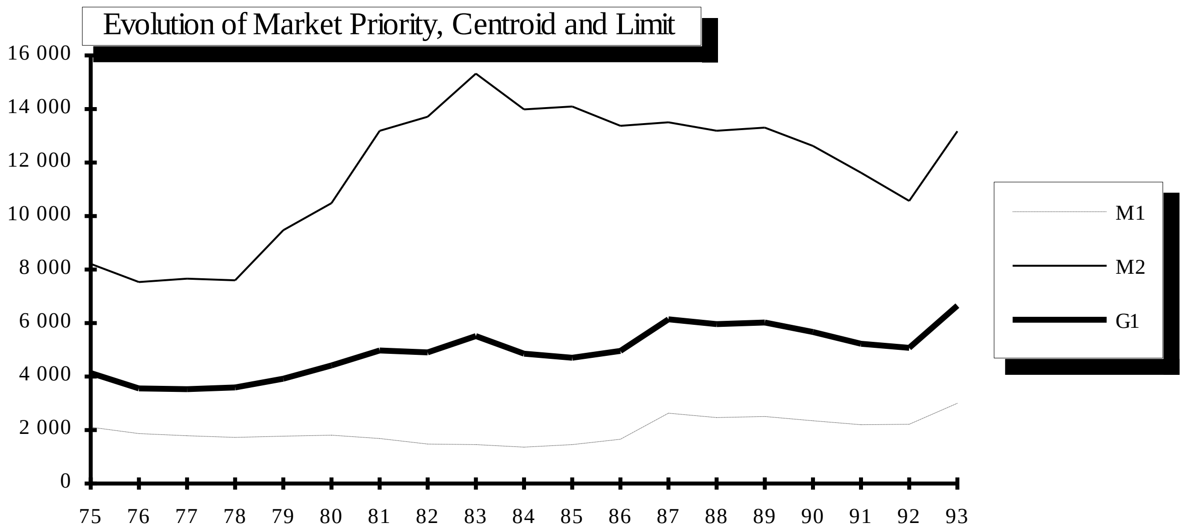
4.5.3 Third Family of Indices



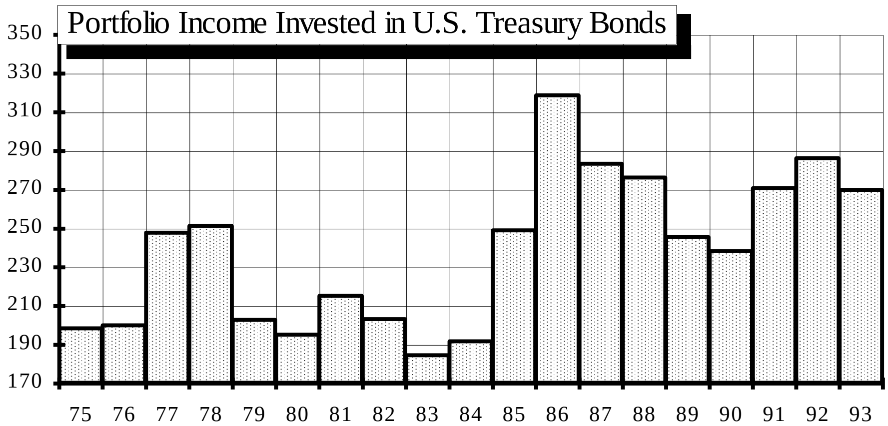
These two curves are less explicit for professionnals. Here we consider another aspect of price, in the sense that we concentrate more on the evolutionary aspect of the program price structure weighting than on the fact that the market under or over evaluates the risk. Here, the point is to focus on actor behavior whether or not an incident occurrs and consequently whether there exists or not memory, etc.... They tend to provide an indication of the homogeneity of theoretical pricing with regard to observed evaluation. When the indicator value is close to zero, as for example in 1992, both pricings tend towards an equilibrium.
But, we are going to examine more closely with the next figure which provides exactly the same information but more explicitly for professionals.



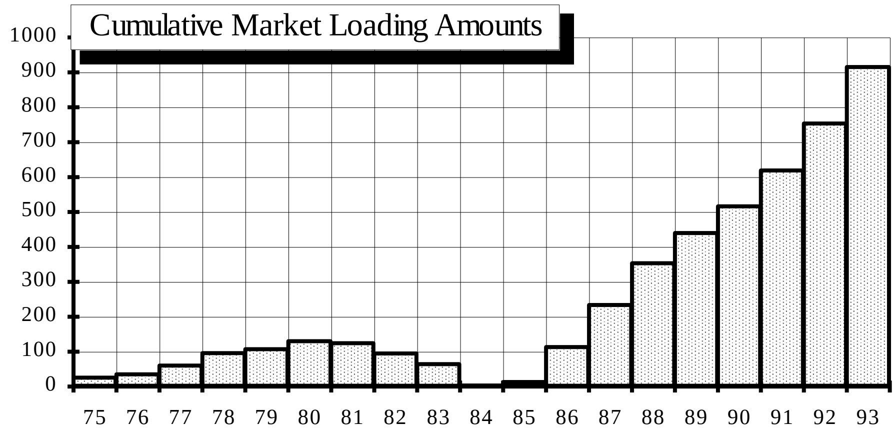
We can see that 1992 is the year of the closest match of theoretical and observed pricing ; it will be noticed that 1975 is also an outstanding year. Fluctuations offer several interpretations : they may show an imbalance in the main program structure between high and low layers. This index can best be explained in the light of the market observed priority, centroid and limit evolution (figure below). Indeed, they provide an indication on the simultaneous evolution of prices on the different levels of the programs. Thus, we can see for example that during the period of the significant fall in the index I_5 level, in 1978 to 1985, the limit increased, the priority decreased, but the price of the high layers decreased ; this being visible thanks to the observed centroid evolution. In contrast, for 1975 we can see a relatively significant gap between layers with at the same time a priority slightly under $2 000 M^{ios} , and for the years 1992 and 1993 a clear increase in prices and in priority and limit levels.



4.6 Global Income Analysis over the Period



In this part, we examined the studied portfolio income evolution in order to provide another time dimension of market evolution and to present a correspondance between market prices and reinsurance demand. Thus suppose that reinsurance premium incomes have been invested in U.S. Treasury Bonds from the effective date to 1992 ; 1993 income was updated in 1992 at the same rate. We can notice here that during the period 1981 to 1984 the demand was very low and it suddenly rose sharply in 1985 that is two years after Alicia and as with any market prices followed. This is currently analysed as the market actors' risk appreciation.
It is then possible to see if there have been cumulative profits or losses over the studied period in regard to incurred risks. The underlying idea here is to highlight the possibility of risk commitment mutualisation in the medium term. Another idea consists in quantifying the eventual losses in case of total reinsurance pure premium consumption.



The considered unit is one million U.S. dollars.
This histogram represents the cumulative amounts reserved for risk compensation by the market when assuming that our chosen portfolio is representative of the whole nationwide market, giving an indication of its evolutionary shape. We deduce that from 1983 reinsurers are at break-even point with regard to their commitments. Note that this result takes into account cumulative profits since 1975. In another way, from 1986 their situation is restabilized becoming definitively profitable from 1992. But we must moderate this analysis because it does not consider whether or not claims occurr. It first states that the pure premium is totally consumed, for the pricing is right and it only considers compensation for extraordinary risks.

5 Modelisation and Econometric Analysis

Recapitulatory table of important catastrophe event occurrence

Year	**1974**	**1976**	**1979**	**1980**	**1983**	**1985**	**1986**	**1988**	**1989**	**1992**
Important Claim (MUSD)	Vent 1 430		Frederic 1 480		Alicia 1 131				Hugo 4 411	Andrew 16 000
Hurricanes number, if greater than 2		2	2	2		6	2	3	5	2
Hurricanes number if greater than 3						6		3	5	

The claim amount influence on the index evolutions can be judged from this table.

We only provide here the econometric analysis for two indices I_1.1 and I_3.1. other analyses being similar both in terms of methodology and of results, it does not seem useful to emphasize this point ; indeed the pursued aim is to provide an index expression taking into account both past price level and occurring claim effect on this index. The time series being short (19 data) it is important to notice that the econometric analysis presented here is provided as a trend indicator instrument and not as a classical forecasting model necessitating much longer series. Nevertheless, the year n price index is a price level indicator for this specific reinsurance market of the same year. And in order to introduce this tool into reinsurance company underwriting strategies it is absolutely necessary to provide the most accurate econometric study within the bounds of presently available data.
The series length imposes an elaboration of a theoretical model with few parameters. The chosen methodology consists in firstly making for each index a time series classical analysis and secondly in attempting to measure the impact of exogeneous intervention variables.

We have considered the three following intervention variables :

- "One claim amounting to over one billion dollars updated to mid 1992 occurred during the year n" $\{S_n \geq 1 \ B^{ion}\}$
- "Two hurricanes occurred during the year n" $\{N \geq 2\}$, where $N = \sum_{j=1}^{4} N_j$
- "At least three hurricanes occurred during year n" $\{N \geq 3\}$.

The equity and the financial market rate have not been integrated in this study, but will be at a later stage.

For this length of series the usual software for time series analysis with introduction of interventional variables does not permit the revealing of these variable eventual impacts on the series. We had thus to go further with the research to reveal the eventual effect. The employed method consists in detecting the dates where the standard error between the observed and the forecast values for the same date is far more important than for the others. In both cases the forecast has been done with the horizon two. The attempt of an ARIMA modelisation by an auto-regressive model of the greater order as possible does not permit the testing of any intervention variable effect. This is justified by the fact that one out of five terms of the series includes the intervention variable effect and so the ARIMA modelisation integrates these interventions in the main structure of the series.

5.1 Analysis and Modelisation of the Evolution of Index I_1.1

Briefly, we could say that the time series modeling offers the choice between a two order auto-regressive model and a one order auto-regressive model. We have selected here the first order model over the second inspite of an explained variance part of 81% in the second case against 73% in the first case. Indeed, the intervention phenomena are integrated by the second order modelisation. Thus it does not enable the explicit inclusion, in the model, of the intervention variable. Though, in the case of first order modelisation data seems to allow the detection of an intervention variable influence. We are able to detect the impact of the intervention variable $\{S_n \geq 1 \ B^{ion}\}$ two years later. We note on the contrary that it seems probable that the phenomena occurring two years consecutively are integrated in the modelisation AR(1). Indeed, this occurrs three times over the 19 data series and justifies the impossibility of observing the influence of past year frequency on price levels. It is worthwhile to note that the impact of the intervention variable in 1979 cannot be taken into account. This can be interpreted as a change which intervenes after 1980 in treaty pricing and made at the begining of the period studied. From 1980, the impact is observed. Before any modeling on the data itself we establish that if there is an intervention at the date n the index value drops slightly in n+1 even though the n+2 value becomes greater.

The obtained modeling follows :

$$I_n = I_{n-1} - 0.02 * \mathbb{1}_{\{S_{n-1} \geq 1 \ B^{ion}\}} + 0.03 * \mathbb{1}_{\{S_{n-2} \geq 1 \ B^{ion}\}} + \varepsilon_n$$

where

(ε_n) denotes a zero mean white-noise process with 0.052 variance (i.e.$(\varepsilon_n) \sim WN(0; 0.052)$)

and, the constant being insignificant.

Data transcribes a two year delay in the market price repercussion from great claims occurring. This can be explained by two points : the delay in the claim amount estimation which takes more than one year, and the delay of both awareness and reaction of the market actor after a great claim occurrence. On the oher hand, the model cannot presently integrate the acceleration in execution recently observed in the market. Moreover it could be interesting to observe in the future the two year successive occurrence of one claim of an amount higher than one billion dollars of mid-1992.

5.2 Analysis and Modeling of the Index I_3.1 Evolution

In this case, the time series analysis only offers the first order auto-regressive modelisation with a 64% part of explained variance. As well as previously seen, the intervention variables being invariable by definition they are once more three times distant of one year. In a short auto-regressive modelisation these factors are in part integrated. Actually, in this case also we only state the impact of the variable $\{S_n \geq 1 \text{ Billion}\}$ whose interventions are distant in time.

The obtained modelisation is thus close to the previous one :

$$I_n = 0.013+0.98\, I_{n-1} - 0.04*1\!\!1_{\{S_{n-1} \geq 1\, B^{ion}\}}) +0.1 *1\!\!1_{\{S_{n-2} \geq 1\, B^{ion}\}})+\varepsilon_n$$

where $(\varepsilon_n) \sim WN(0;0.1264)$.

The practical use and the interpretation are then similar apart from the fact that, in this case, the effect of the second order delay stands out more clearly than in the modeling of the index I_1.1.

6 Conclusion

Assuming that theoretical prices are reliable, the index evolution curves underline a very clear convergence of all the indices, thus suggesting 1992 as the pricing year best matched to the claim amounts distribution chosen by this study.
The question now is does the chosen distribution which lies on updated observations on the period 1950 to 1992, undervalue reinsurance price as suggested by other kind of studies relying on simulation methods with meteorological parameters ? We do not exclude then, the fact that from the probable meteorological mildness of the last thirty years the price observed in 1992 in fact still minimizes the risk price (SCOR 1993, SIGMA 1993, Applied Insurance Research 1992).

Regarding reinsurance market cycles the econometric analysis shows that the price level for one given year is strongly correlated with that of the previous year. On the other hand, the major claims occurrence creates leverage on prices and brings about their increase with a delay either one or two years. This effect has several causes which have been measured solely through prices in the model proposed by this paper. One example is the psychological reaction of the market actors. Indeed, the important claim occurrence provokes a new risk judgment. These specific periods are transcribed by a desire for a new risk cost appreciation legitimized by its previous underevaluation. We can notice that this strategical structure becomes apparent from the hurricane Alicia occurrency date (1983). This behavior is presently strengthened by the market financial capacity contraction. It is appropriate to note that this economic upward trend is countered by the effects of the greater permeability of information thus contributing to a more efficient management and tending to cancel out the snowball effect. Their early signs are ascertained in the graphs by perceiving a one instead of two year reaction delay in the present upward trend. And considering the information permeability we must take into account a probable hardening of competition. Anticipations of this situation can be integrated to these models by simulation methods on premiums rates and this with a view to forecasting. It seems to us just as necessary to keep active this kind of model in order to obtain a more econometrically significant series. More immediately each passing year will thus permit discussion of the theoretical purposes of this paper. Let us now focus on the practical applications and possible extensions of the paper. The more immediate application is the design of a tool adapted for the underwriting of these treaties. As demonstrated by this study, it is clear that it is strongly in the actor's interest to enter the market during the two year exercise after a major catastrophe. In the same way, a reinsurer if he can, must not leave the market during these two years and this proposition also holds for the insurance company because of the direct premium general upswing.

We shall conclude by indicating two extensions to the study. The first concerns its generalisation to proportional reinsurance in order to create a sharper tool permitting the choice of the most profitable method (proportional or non-proportional) in strategic terms and consequently offering an indication for financial capacity allocation. The second extension consists in the creation of an arbitration tool permitting the weighting of the sometimes antagonistic interests of a company in order to reconcile all the criteria relating to yield and image.

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